# Data, Reduced Order Models and Artificial Intelligence for Digital Twin

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**Abstract.** An increasing number of disruptive innovations with high economic and social impact are characterizing the digitalization processes occurring in the last years. Despite the benefits of these developments, their speed and extent are limited by the available simulation technologies to handle complex models. The key idea of Digital Twin (DT) lies exactly in the creation of a digital counterpart of the physical asset able to replicate its behaviour. The necessity of a continuous and fast interaction between the physical and digital environments through background simulations has led to the introduction of Reduced Order Modeling (ROM) and Artificial Intelligence (AI) as crucial technologies to speed up the model execution, while maintaining good level of accuracy but with a reduced number of degrees of freedom. In this way, DT can be applied to new complex emerging fields, providing also more accurate information on the real counterpart, thanks to the presence of these techniques.

**Keywords.** Digital Twin, Reduced Order Model, Data Analysis, Artificial Intelligence, Real Time Computing

## Introduction

Reduced Order Modelling is a quickly emerging field in applied mathematics, computational science and engineering for speeding up Numerical Simulations. The growing demand of efficient computational tools and real time computations, coupled with the presence of parametric formulations and uncertainty quantification in the models under consideration have led to the necessity of a computational collaboration between High Performance Computing(HPC) and Reduced Order Methods(ROMs)[14]. The former deals with high order models, characterized by very expensive and time demanding simulations requiring HPC facilities. ROMs enable to overcome these difficulties by constructing a reduced version of the full model, faster than the previous one. In this way, real-time input-output evaluations are possible without requiring demanding resources.
Artificial Intelligence (AI), Deep Learning (DL) and Machine Learning (ML) are revolutionizing the field of Computational Science bringing high generalization capability. Neural networks, with their capacity to learn and approximate complex patterns, have emerged as powerful tool in several contexts. Therefore, integrating neural networks into ROMs offers a powerful approach to approximate complex systems efficiently and accurately. DL plays an important role also in managing the huge amount of data at disposal nowadays. This vast amount of data, constantly generated by several sources , requires advanced tools and techniques for storage, processing, and analysis to uncover valuable insights and drive decision-making. The use of ML techniques coupled with HPC allow the mana-



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gement of Big Data and a real-time analysis of the continuous flow of such data coming from various sources such as simulations, sensors, etc. They also can ensure an estimation of the uncertainty on such data to ensure data veracity and the capacity of the extraction of knowledge and significant patterns from the available data.
Digital Twins (DTs) represent thus a powerful tool for integrating ROMs, HPC, AI, Big Data and Uncertainty Quantification. By creating a virtual replica of physical systems, DTs enable real-time monitoring and simulations, handled through the application of HPC and ROMs. AI can then analyze vast amounts of data for predictive maintenance and optimization. The synergy of these technologies with Big Data ensures more accurate and efficient decision-making, enhancing the performance and reliability of systems across various applications.

## 1. An integrated paradigm: Digital Twin

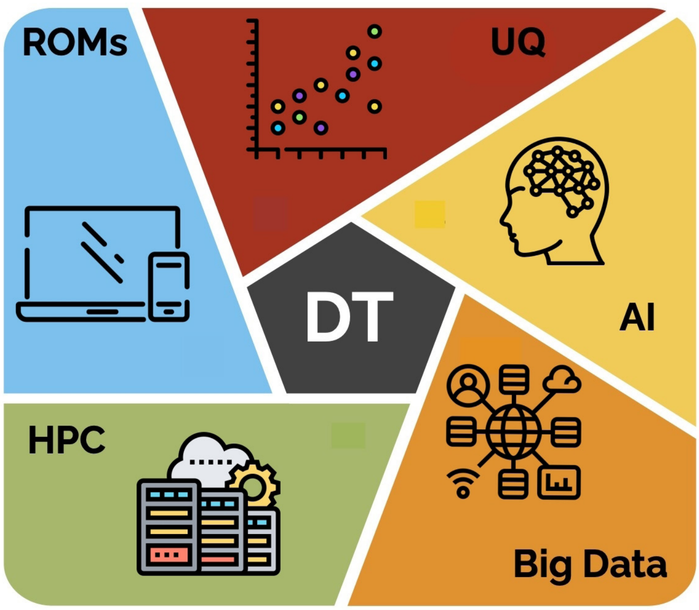



Fig. 1
An integrated paradigm: Digital Twin

A Digital Twin (DT) represents a virtual replica of an object or phenomena, which can be used to gain improvements in the physical part. This is obtained through an automatic flow of data from the physical object to the digital one, making DT a fundamental support tool for the entire life-cycle of an object.
As represented in Figure 1, DT represents a paradigm integrating several elements:
1. Big Data: Data are the basis for the construction of such models. Nowadays, IoT devices represent a great source of data due to their increasing presence in everyday objects, factories, etc.
2. High Performance Computing (HPC): This huge amount of data needs to be handled through suitable infrastructures able to capture the underlying structure and information

contained inside them. This amount of data also represents the input for some mathematical model describing the problem under consideration. These models are often characterized by a high number of parameters due to the complexity of the modeled problem. HPC is thus fundamental in storing Big Data and running complex simulations.

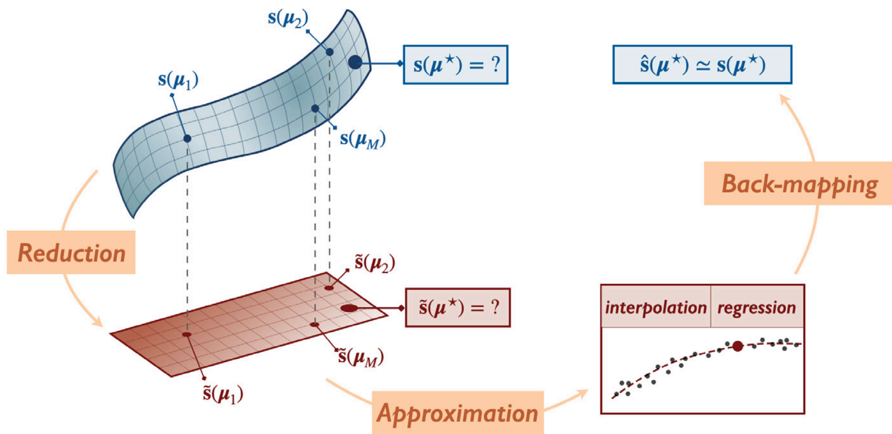



Fig. 2 Schematic representation of the reduction process for ROMs

3. Reduced Order Models (ROMs): Due to the complexity (high number of parameters) of models, simulations are slow and very expensive from the computational point of view. To be applied in an industrial environment, simulations need to be real-time and with a low computational cost. ROMs represent thus an approach to handle this problem, providing models reduced in the number of parameters but with high accuracy. The goal of these methods is to detect the most important direction and information in the parameter space, to perform a reduction based on them. An approximated solution can then be constructed through an interpolation/regression process.
4. Artificial Intelligence (AI): AI techniques, such as machine learning and neural networks, enhance simulations by learning complex patterns from data, improving accuracy and efficiency. These methods can also be integrated in ROMs to enable better approximations of complex systems.
5. Uncertainty Quantification (UQ): UQ refers to the process of assessing and managing uncertainties that arise from both data and the mathematical models used to interpret that data. It is thus essential for improving the reliability and applicability of models in several fields.

## 2. Reduced Order Models enhanced by Deep Learning and Scientific Machine Learning

Artificial Intelligence represents a tool increasingly integrated in several fields. In particular, it can be employed to enhance classical (data-driven) ROM techniques for Computational Fluid Dynamics. Some examples include the following methods:

- Approximation in Reduced Order Model: ANNs are introduced to extract information coming from data and to approximate the functional relationship between input parameters and output solutions. POD-NN and AE-NN represent possible techniques developed for this purpose [1, 2, 3, 4, 8]. Neural Networks can thus be adopted to improve the accuracy of a POD-based model by improving generalization and providing a multi-fidelity perspective.

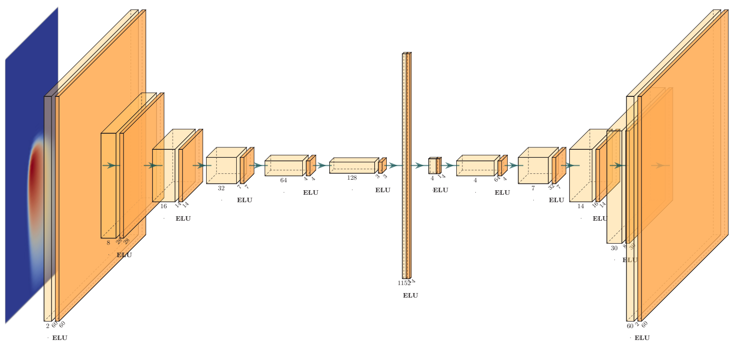


Fig. 3
A convolutional Auto-encoder

- Auto-encoders for dimensionality reduction and manifold learning: Autoencoders are included in data-driven methods to learn a latent representation of the dynamical system. To include the knowledge about the physics of the model, we can couple a non-linear manifold method [6] with hyper-reduction achieved through reduced over-collocation and teacher–student training of a reduced decoder [5].
- Reduction in wide parameter space by means of deep learning parameter domain decomposition: A Deep Learning framework can be developed to generalize the evolution of a system over initial conditions in an extended parameter space. In [7] a two stage memory aware ANNs model order reduction approach has been constructed to deal with this problem. Curse of dimensionality is initially tackled by partitioning the parameter space and averaging ROM solutions. Then Long-short term memory network (LSTM) are coupled with Convolutional Networks (C-LSTM) for extracting temporal correlations.
- Generative models to quantify model uncertainty: As described in the previous section, UQ represents an important element to keep into account. In [13], an approach integrating generative modeling with UQ is presented, where the former is employed to learn probability distributions on the data and to capture the intrinsic characteristics of the original ones. A priori UQ quantification can thus be done exploiting this probabilistic approach.

## 2.1 Machine Learning pipeline

Given a problem (e.g. fluid dynamics, stochastic PDEs), a pipeline can be defined to include Machine Learning in the solution of the corresponding PDEs. The first step coincide with the generation of data for the problem under consideration. Hence, high fidelity simulations can be performed or scattered data from the domain can be determined. Once the problem and the data have been determined, a ML model can be built, using for example

Neural Networks, POD with Interpolation, Neural Operators, etc.. The techniques listed above represent possible solutions for the construction of ML models. The last step coincides then with the optimization of the model using, for example, Supervised Learning, Physics-Informed losses and the gradient descent method.
An example of integration of Deep Learning into PDEs is represented by Physics-Informed Neural Networks (PINNs) [9,10, 11,12], an optimization technique to compute solution of differential equation using Neural Networks
In this framework, PDEs are solved by including physical laws and symmetries in Neural Networks. PINNs represent thus an approach that balances data and physical knowledge, coming from ROMs simulations, in order to build a truthful and reliable ML model.

Fig. 4
Machine Learning pipeline

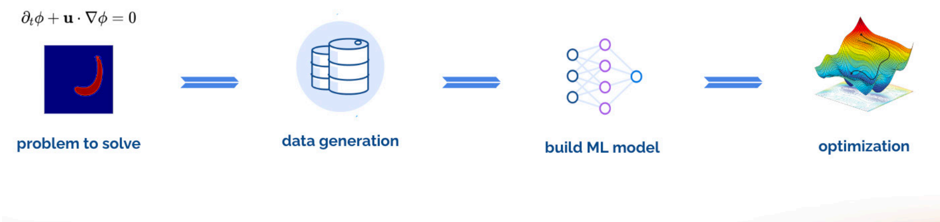
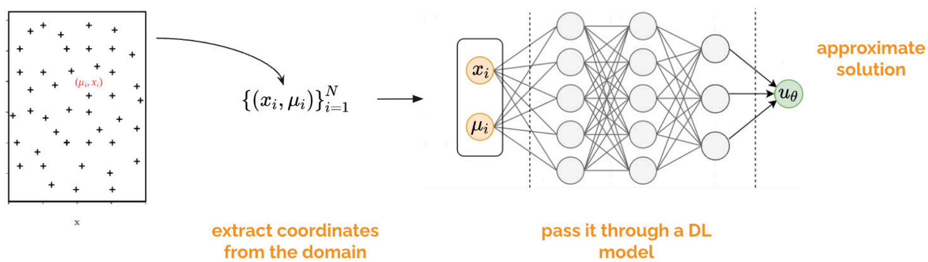



Fig. 5
PINNs pipeline




## 3. Applications of Digital Twin

As explained in the previous sections, a digital twin is the virtual replica of a physical asset. It means that it is possible to realize a digital twin in very different fields. It could be a DT of product or processes which means, for instance, a manufacturing asset or manufacturing plant; but it can be translated into a DT of an environment or entire processes.
We developed three Digital Twins, more specifically three Live Demos, related to the Competence center Industry 4.0 of the north-east of Italy, SMACT Odyssea. Those are three useful and tangible examples to understand the potential of the tool: one is a Smart Hybrid Energy Management System with Wartsila connected to hybrid engines of vessels; the second one is a Heavy-Duty Machining Processes Optimisation with Danieli Automation and the last one, still under development, is related to an aspiring hood with Electrolux.
We do mention also the activities of Spoke 9 of iNEST, Interconnected Innovation Ecosystem of North-East of Italy dedicated to computational technologies for digital twins: including infrastructures, services, networks, communities asd well as medicine. The main concept which stands behind this innovative tool is the flux of information coming from

the real asset going to the virtual replica and come back to the real one: it is crucial to obtain real time information necessary to take appropriate actions in case of necessity.

## 4. Conclusions

Digital twins are revolutionizing industries by offering virtual replicas that simulate physical assets, processes, or systems in real-time. This technology enables proactive maintenance, performance optimization, and efficient resource management. Data, Reduced Order Modeling, Control, Optimization, Machine Learning, Artificial Intelligence and Uncertainty Quantification play a key role in the definition of DTs, leading to the need of a better integration to create this new parametrized, reduced and coupled paradigm. Applied Mathematics is thus fundamental in the advances of Science and Engineering, becoming a propeller for methodological innovation and technology transfer.

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